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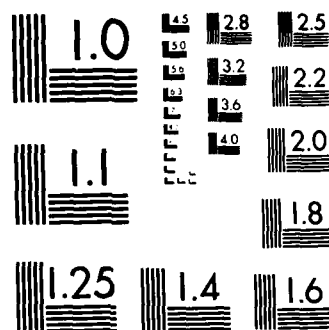
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MOTION AND STABILITY OF SATURATED SOIL SYSTEMS  
UNDER DYNAMIC LOADING

Ranbir S. Sandhu  
Department of Civil Engineering

For the Period  
February 1, 1984 - January 31, 1985

DEPARTMENT OF THE AIR FORCE  
Air Force Office of Scientific Research  
Bolling Air Force Base, D.C. 20332

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19. (Continued)

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Variational formulations of Biot's theory were developed to construct a basis for alternative finite element approaches to the problem. For nonlinear problems, incremental equations were developed and variational formulation attempted allowing only material nonlinearity.

In saturated soils subjected to dynamic loads, depending upon permeability and pore geometry, a part of the water would possibly be trapped and move with the soil rather than relative to it. This mass coupling effect was examined in a parametric study. It was found that soil displacements are not sensitive to the degree of coupling but the pattern of pore pressure in the time domain could be affected significantly. Further studies in this direction are apparently needed.

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Annual Report to the  
Air Force Office of Scientific Research

MOTION AND STABILITY OF SATURATED  
SOIL SYSTEMS UNDER DYNAMIC LOADING

Principal Investigator: Ranbir S. Sandhu

Department of Civil Engineering

Period of Performance: February 1, 1984 to January 31, 1985

\* \* \* \* \*

THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION  
1314, KINNEAR ROAD, COLUMBUS, OHIO 43212

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ABSTRACT

In the second year, review of the assumptions underlying various theories of interacting continua was completed. An important finding was that the notion of the mixture as a continuum in motion is inadmissible except in the case of no relative motion between the constituents. Liquefaction of soil is primarily associated with relative motion of soil and water.

The so-called 'engineering approach' to liquefaction was determined not to be worth pursuing further because it cannot be extended to two and three dimensional situations and is not quite reliable.

Finite element implementation of Biot's theory was studied in respect of effectiveness of the popular time integration schemes as well as spatial discretization for one and two-dimensional wave propagation. The results showed that the conventional time-domain integration procedures which are quite effective for single material problems are not reliable for saturated soils. The numerical results were found to be quite sensitive to the choice of time-domain integration parameters. Further work on developing reliable integration techniques is needed.

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## 1. INTRODUCTION

Dynamic loading on fluid-saturated geological deposits results in changes in the fluid pressure as well as the stress field in the solid matrix. This phenomenon, in some cases can lead to instability of the soil matrix resulting in soil liquefaction and, also, the energy dissipation associated with the relative oscillatory motion between the fluid and the solid could introduce attenuation of the propagating wave. This effect would be in addition to the energy dissipation in any inelastic deformation of the soil matrix. Inasmuch as the transmissibility of motion through the soil layer depends upon the soil characteristics, the soil deposit may act as a selective filter/amplifier. The importance of transmission characteristics and stability of the soil matrix in the study of earthquake hazard and risk of structural damage has long been recognized. For seismograph installations located on soil deposits, the records include local site effects which have to be allowed for. Methods of analysis, including numerical procedures, have been developed to calculate the surface motion response and histories of stress within the soil layers. However, the currently available procedures are based on several simplifying assumptions and would not always be reliable. For instance, the spread of pore water pressure in a deposit and development of failure cannot be correctly determined by the current methods. There was evident need to develop more realistic models of mechanical behavior of fluid-saturated soils and to implement these models in effective computational procedures.

A two year program of research was initially approved by the AFOSR to systematically cover the items needing investigation. This has been since extended to a four year effort.

## 2. RESEARCH OBJECTIVES

The objective of the continuing research program is to critically evaluate the current methods of analysis in the light of recent developments in theories of interacting continua and to develop computational techniques for dynamic response based on thermodynamically and physically consistent modeling of the fluid-saturated soil systems. The theories of interacting continua, in general, regard a fluid-saturated soil as a multi-component mixture (superposed continua). This approach has been successfully implemented in analysis of static and quasi-static response of saturated soils. Several investigators have tried to extend this approach to the dynamic case. However, there have been deficiencies in realistic simulation of behavior of saturated soils. The purpose of the current research program is to critically review the theoretical basis of equations governing behavior of soil-water mixture under dynamic conditions, properly allowing for dynamic interaction between soil and water and ensuring thermodynamic consistency in postulating constitutive relationships. The theoretical development is to be the basis for effective finite element computer programs, incorporating recent improvements in coding to ensure optimal combination of solution accuracy and economy.

In the first two years, review of currently used models and selection/development of a formulation appropriate to the problem was to be completed. Work on development of numerical/computational models to implement various theoretical concepts was to be started and various computational approaches investigated. The theoretical development was to be verified in a laboratory program. Model systems, for which boundary conditions are well defined, were to be subjected to known dynamic excitations. Instrumentation capable of measuring the time rate of pore pressure increase at a number of locations within the sample as well as model deformations and accelerations, was to be utilized. Comparison of the observed model response with that predicted by the mathematical/computational model was to provide the basis for revision and updating of the theoretical/computational models. The continuing research effort will extend and refine the theoretical models, allow for compressibility of soil as well as water, admit nonlinear constitutive relationships, large deformations and mass coupling effects.

### 3. STATUS OF THE RESEARCH

Adequate progress has been made. Available mathematical models of the phenomenon have been reviewed. Fourteen research reports/papers have been prepared/completed. These include four publications in Conference proceedings. In addition, some results of the research were presented at two additional conferences.

Theories of interacting continua were reviewed carefully in the context of their application to the dynamics of saturated soils. The assumptions on which competing theories are based were identified and evaluated. The notion of the mixture as a continuum in motion has been found to be inadmissible except in the case of no relative motion between the constituents. This is an important finding because liquefaction is primarily caused by the relative motion of soil and water and, therefore, a correct theory of liquefaction should not include this assumption.

Finite element methodologies have been explored. Computer programs based on different approaches were developed to select a reliable procedure for predicting the behavior of the material in the laboratory set-up.

The so-called 'engineering approach' to liquefaction was studied. It was found that this approach cannot be extended to two and three dimensional situations and is not quite reliable. It is not worth pursuing any further.

Finite element methodologies have been explored. Computer programs based on different approaches were developed to select a reliable procedure for predicting the behavior of the material in the laboratory set-up.

Finite element implementation of Biot's theory was studied in respect of effectiveness of the popular time integration schemes as well as spatial

discretization. Three different finite element strategies were used to cover the cases of one and two-dimensional wave propagation. Both bilinear and biquadratic interpolation schemes were implemented. The results for one-dimensional analysis, for which exact solutions are available, showed that the conventional time-domain integration procedures found to be quite effective for single material problems are not reliable for the coupled problem of dynamics of saturated soils. For two-dimensional cases, no exact solutions are available. It would be foolish to expend a lot of time and effort in developing computer programs for two-dimensional analyses before the solution process has been refined sufficiently to give satisfactory agreement with the exact solution for the one-dimensional problem. The results should not be sensitive to the choice of time-domain integration parameters. Also, as the time step size is reduced, the approximate solution should converge. It is necessary, therefore, to investigate the time integration procedures carefully to identify and implement stable schemes relatively insensitive to the parameters selected by the researcher. Work on both single-step and multi-step schemes for this purpose was started. Refinement of the models will be pursued further in the next year.

Variational formulations of Biot's theory were developed to construct a basis for alternative finite element approaches to the problem. For nonlinear problems, an incremental approach is necessary. The equations governing this case were developed and variational formulation of the incremental problem attempted. At this time only material nonlinearity is being considered.

In saturated soils subjected to dynamic loads, depending upon permeability and pore geometry, a part of the water would possibly be trapped and move with the soil rather than relative to it. This mass coupling effect, originally postulated by Biot, was examined in a parametric study. It was found that soil displacements are not sensitive to the degree of coupling but the pattern of pore pressure in the time domain could be affected significantly. Further studies in this direction are apparently needed.

In the first year of the research effort, work was started on setting up laboratory experiments for verification of the theory. Necessary equipment and instrumentation was acquired. During the second year, the equipment was set up, peripherals for laboratory testing developed and the instrumentation calibrated. Work on laboratory experiments was held up for some time because of the non-delivery of an essential item in the data-acquisition system. The entire system was built up during the year and actual experimental investigation started. Sand for the experiments was acquired, means for placement of sand in the experimental box, insuring saturation with de-aired water, were developed and properties of the sand carefully analyzed to obtain the basic data needed by the mathematical models. The program of laboratory verification of the theory was started and is continuing.

#### 4. LIST OF PUBLICATIONS

In the second year, the research program resulted in the following publications.

1. Ranbir S. Sandhu, Baher L. Aboustit, and S. J. Hong, An Evaluation of Finite Element Models for Soil Consolidation, Geotechnical Engineering Report No. 11, OSURF 763420/715107-84-2, AFOSR Grant 83-0055, April 1984.
2. Mahantesh S. Hiremath, and Ranbir S. Sandhu, A Computer Program for Dynamic Response of Layered Saturated Sands, Geotechnical Engineering Report No. 12, OSURF 763420/715107-84-3, AFOSR Grant 83-0055, June 1984.
3. Ranbir S. Sandhu, Mechanical Behavior of Fluid Saturated Soils, Geotechnical Engineering Report No. 10, AFOSR Grant 83-0055, April 1985.
4. Ranbir S. Sandhu, Baher L. Aboustit, and S. J. Hong, Response of Saturated Soils To Dynamic Loading, Geotechnical Engineering Report No. 13, OSURF 763420/715107-84-4, AFOSR Grant 83-0055, June 1984.
5. Ranbir S. Sandhu, Baher L. Aboustit, S. J. Hong, and M.S. Hiremath, A Computer Program for Consolidation and Dynamic Response Analysis of Fluid-Saturated Media, Geotechnical Engineering Report No. 14, OSURF 763420/715107-84-5, AFOSR Grant 83-0055, June 1984.
6. Ranbir S. Sandhu, "On Constitutive Equations for Fluid-Saturated Porous Solids", Proceedings, Engineering Foundation Conference on "Compressibility Phenomena in Subsidence", Henniker college, Henniker, New Hampshire, July 29, 1984 to August 3, 1984.
7. Ranbir S. Sandhu, Baher L. Aboustit, and S.J Hong, "Finite Element Analysis of Flow and Deformation in Saturated Soils", Proceedings, Second Symposium on Interaction of Non-Nuclear Munitions with Structures, Panama City, Florida, April 1985.
8. Ranbir S. Sandhu, S.J. Hong, and Baher L. Aboustit, "Analysis of Response of Saturated Soil Systems Subjected to Dynamic Loading", Proceedings, Second Symposium on Interaction of Non-Nuclear Munitions with Structures, Panama City, Florida, April 1985.
9. Mahantesh S. Hiremath, and Ranbir S. Sandhu, "Application of Theories of Mixtures to Behavior of Fluid-Saturated Deformable Porous Media", Proceedings, Second Symposium on Interaction of Non-Nuclear Munitions with Structures, Panama City, Florida, April 1985.
10. Ranbir S. Sandhu, and S.J. Hong, An Investigation of Mass Coupling in Dynamic Response of Fluid-Saturated Soils, Geotechnical Engineering Report No. 15, OSURF 715729-85-1, AFOSR Grant 83-0055, April 1985.

11. Ranbir S. Sandhu, and S.J. Hong, Variational Formulation of Dynamics of Fluid-Saturated Soils, Geotechnical Engineering Report No. 16, OSURF 715729-85-2, AFOSR Grant 83-0055, April 1985.
12. Ranbir S. Sandhu, and Mahantesh S. Hiremath, Variational Formulation of Dynamics of Nonlinear Fluid-Saturated Soils, Geotechnical Engineering Report No. 17, OSURF 715729-85-3, AFOSR Grant 83-0055, April 1985.
13. Ranbir S. Sandhu, and Vincent E. Amato, Characterization of Silica Sand for Liquefaction Experiments, Geotechnical Engineering Report No. 18, OSURF 715729-85-4, AFOSR Grant 83-0055, May 1985.
14. Ranbir S. Sandhu, Vincent E. Amato and Raymond Kolonay, Preliminary Tests on Liquefaction of Saturated Silica Sand Under Dynamic Excitation, Geotechnical Engineering Report No. 19, OSURF 715729-85-4, AFOSR Grant 83-0055, June 1985.

Items 1, 2, 4, and 5 represent completion of documentation on work done during the first year. These reports have been approved by AFOSR. Draft of item 3 has been submitted to the sponsor. Items 6 thru' 9 have been accepted for publication and reprints will be submitted when available. Items 10 thru' 14 are under documentation and will be submitted for review in the near future.

#### 5. LIST OF PROFESSIONAL PERSONNEL

1. Ranbir S. Sandhu, Professor, Department of Civil Engineering, Principal Investigator.
2. M. Hiremath, Graduate Research Associate, doctoral student.
3. S.J. Hong, Graduate Research Associate, doctoral student.
4. C.C. Chang, Graduate Research Associate, doctoral student.
5. V. Amato, Research Assistant, master's degree student.
6. Raymond Kolonay, Research Associate, Master's degree student.

Mr. Amato is currently a University Fellow and not a burden to the project. However, he has been working on the experimental phase of the research.

#### 6. INTERACTIONS(COUPPLING ACTIVITIES)

Item 6 in Section 4 above was presented at the Engineering Foundation Conference held in July/August 1984. In addition, the following presentation were made to present the findings of research. Items 7 thru' 9 will be presented at the Second Symposium on Interaction of Non-Nuclear Munitions with Structures, to be held at Panama City, Florida, in April 1985. The principal investigator was a member of the organizing committee of an Engineering foundation Conference devoted to compressibility phenomena in subsidence.

Mechanics of saturated soils was highlighted at this conference. In addition to the above-listed presentations which will be published in proceedings of the respective conferences, the following presentations were made in response to invitations from the conference committees:

1. Ranbir S. Sandhu, "Application of Theories of Mixtures to Saturated Soils", Engineering Foundation Conference on "Compressibility Phenomena in Subsidence", Henniker College, Henniker, New Hampshire, July 29, 1984 to August 3, 1984.
2. Ranbir S. Sandhu, "Finite Element analysis of Flow Through Deformable Porous Media", Society of Engineering Science, 21st Annual Meeting, 15-17 October, 1984, Virginia Polytechnic and State University, Blacksburg, Virginia.

A summary of results of the research was made available to the National Research Council, Committee on Earthquake Engineering, Panel on Liquefaction. This panel planned to hold a Workshop at MIT during March 1985. A Seminar will be conducted later in the year.

## 7. NEW FINDINGS

### 7.1 Theoretical Studies.

Three approaches have been applied to analysis of soil liquefaction. These are:

- i. The Engineering Approach
- ii. Biot's Theory
- iii. Theories of mixtures

#### a. The Engineering Approach

This approach, introduced by Seed, uses methods of structural dynamics to solve the problem of shear wave propagation in soils. Essentially, this approach consists of a finite element analysis of the system, under an acceleration history applied to a number of points on the system, to evaluate the stress history. This is followed by a laboratory study of the material behavior under cyclic stress conditions equivalent to those determined from the finite element analysis. This approach was used, with apparent success, by Seed and his co-workers to analyze a number of case histories. Subsequently, the method has been improved to include a periodic updating of material behavior to allow for the strain history as well as pore-water pressure build-up and dissipation. Finn called this the coupled theory of liquefaction. The sequence of occurrences is assumed to be as follows: shearing stresses cause volume changes, volume changes result in pore water pressure changes, pore-pressure dissipation follows, pore-pressures determine effective stresses and effective stresses along with the cumulative shearing strain define the effec-

tive shear modulus to be used for determination of displacements and stresses for the next time step.

Item 2 in Section 4 above contains the details of the methodology described above as well as its implementation in a computer program. The procedure is cumbersome. The physical properties can only be determined as a function of the complete number of cycles of stress at a certain amplitude. Thus, in an explicit type solution scheme, it is possible to reduce the time interval for updates to only as low as the time period of vibration. For this reason it was not possible to generate a convergent sequence of solutions based upon reducing size of the time step. Implicit schemes are expected to be more reliable. However, the results from the explicit and the implicit methods did not agree. Under harmonic loading, the time to liquefaction and the location of the layer that liquefies were correctly determined. However, the acceleration and displacement response calculation is not reliable. Post-liquefaction distribution of pore pressures cannot be correctly determined in this theory. The approach has only been used for one-dimensional wave propagation. It cannot be extended to two and three-dimensional cases. Use of this theory requires considerable experience and "judgement", in addition to extensive laboratory testing program, to get useful results.

#### b. Biot's Theories

Biot's formulation of the coupled problem was the basis of theoretical solutions developed, among others, by Biot, Deresiewicz, Garg, and Chakraborty and of numerical solution procedures developed by several investigators including Ghaboussi, Zienkiewicz, Prevost, and Sandhu. For a statistically isotropic saturated material, the kinetic energy was expected to be quadratic in the velocities of the fluid and the soil and a coupling term was included. For the dissipative case a dissipation function quadratic in relative velocity, was introduced. The value of the coefficient in the dissipation term was related to the permeability coefficient.

Biot wrote the constitutive equations for the flow of a compressible fluid through a porous saturated linearly elastic anisotropic medium assuming the existence of an energy function quadratic in soil strain and change in water content. Garg's formulation can be shown to correspond to Biot's. In Garg's theory, the constants are related to the properties of the constituents and the volume fractions.

While developing finite element solution procedures for the problem, Ghaboussi, following Biot, introduced relative volumetric strain in the formulation. The momentum balance and the continuity equations were written in terms of six displacement components viz. the soil displacements and the relative displacements of the fluid. An arbitrary Rayleigh type viscous damping term was introduced. This intrinsic damping of the soil is in addition to the damping associated with relative motion.

We note several different forms of Biot's theories. The most general form includes body forces, inertia forces and the effects of coupling of the fluid and the soil mass. However, several objections to Biot's formulation have been raised. His work was based on ad hoc postulates regarding description of motion, notions of partial stresses, and the existence of energy and dissipation functions for the saturated mass. Modern theories of mixtures applicable to structured media like saturated soils were expected to provide some answers to the questions regarding applicability of Biot's theories.

In the present research program, Biot's theory was implemented in a finite element computer program following the methods used by Ghaboussi and Wilson. It was noted that Ghaboussi had developed a variational formulation for the problem but for the purpose of finite element analysis he used the Galerkin procedure. Ghaboussi did not allow for the boundary conditions properly and did not consider interelement discontinuities inherent in finite element methods. The analysis assumed all the pore water to move relative to the soil. Actually, a 'mass coupling' can exist, that is, a portion of the water could be moving with the soil. This mass coupling was introduced as an additional variable in the analysis. Items 4 and 5 in section 4 above contain details of the program development and some applications to one-dimensional wave propagation problems for which exact solutions are available. Effect of mass coupling on the dynamic response is the subject of item 11 in section 4. Variational procedures allowing for the boundary conditions and discontinuities across interelement boundaries are discussed in item 12 in the same section.

For time-domain analysis, it is customary to use single step time integration schemes. A popular scheme, used among others by Ghaboussi, is the  $\alpha$  method first introduced by Wilson as an extension of Newmark's method. For dynamics of saturated soils, this procedure appears to be inadequate. A parametric study showed that the numerical results were sensitive to the choice of the parameters of the scheme. It is necessary to examine other possibilities including multi-step methods to develop a reliable procedure. This work was taken in hand.

Earlier work on analysis of soil liquefaction based on Biot's theory has been for elastic soils. Most sands are not elastic. An effort to include nonlinear material properties in the analysis was started. The procedure will be modular so that a variety of models can be selected. It is proposed to implement the 'cap model' and Lade's elastic plastic model in the present effort. Of course the model will have to reflect the properties of the material to be used in the laboratory experiments.

### c. Theories of Mixtures

Review of theories of mixtures including their possible relationship with mechanics of saturated soils and liquefaction phenomena was completed. Biot's theory is based on several ad hoc assumptions. It was necessary to establish



their reasonableness or otherwise by reference to fundamental principles of mechanics. Truesdell and Green's theories represent the two most important points of view examined. Other formulations were studied in the context of the principles laid down by Truesdell and Green.

Truesdell wrote the local form of the equations of balance of mass, linear momentum, angular momentum, and energy for the constituents and for the mixture. He introduced the notion of a diffusive force and wrote the local form of an entropy inequality for the mixture. In developing a rational theory of mixtures, Truesdell laid down the following principles:

1. All properties of the mixture must be mathematical consequences of properties of the constituents.
2. So as to describe the motion of a constituent, we may in imagination isolate it from the rest of the mixture, provided we allow properly for the actions of the other constituents upon it.
3. The motion of the mixture is governed by the same equations as is a single body.

To ensure satisfaction of these principles, the total stress had to be defined so that it did not equal the sum of partial stresses in the constituents. Green proposed an alternative approach assuming that the total stress must equal the sum of partial stresses in the constituents. A deficiency in the theory was corrected to admit interacting partial stresses in the constituents which may not contribute to the equations of motion or the total stress. Green considered the concepts of stress, heat flux, and energy supply to be primitive to each constituent and to the mixture as a whole as well. It was proposed that the total stress and the total heat flux for the mixture should equal the sum of the corresponding quantities for the constituents. Other theories of mixtures of immiscible constituents, applicable to the problem of fluid-saturated soils, including the so-called volume fraction theories have been proposed.

It is important to note here that the mixture cannot be regarded as a continuum, the particles of which are constructed by superposition of constituent particles, except in the case of no relative motion between the constituents. In studying dynamic behavior of sands leading up to failure by liquefaction, that case is of no interest. Atkin pointed out that the mixture having a mass density calculated as the weighted average of the constituents cannot be associated with a material in the physical sense. The mixture consists only of centers of mass and is not a set of particles. Sandhu pointed out that the mixture defined above does not satisfy the impenetrability postulate except in the case of no relative motion. It should, therefore, be kept in mind that the total density is a purely mathematical entity having no physical interpretation except in a special case. Many investigators introduce a material rate for the mixture. However, this material rate has no

particular physical significance because the rate is executed not on a material particle but on a center of mass.

Several approaches have been used to describe the motion of the constituents of a multicomponent mixture. One is to refer the motion of each constituent to its reference configuration. Another is to refer the motion of all constituents to the reference configuration of one of them. Yet another is to refer all motion to the current configuration which is the same for all constituents. Superposition of relative diffusive motion of the constituents upon the mean motion of the mixture as a whole is also used. For a binary mixture of a solid and a fluid, some investigators, following Biot, describe the motion of the solid with respect to its reference configuration but the motion of the fluid is described as relative to the solid. Another procedure refers to a material region consisting of the same set of particles of one of the constituents so that the bounding surface of this constituent varies with time.

Truesdell postulated equations of balance of mass, linear momentum, moment of momentum and energy such that the form of the equations was the same for each constituent and for the mixture. Apparently, the existence of the mixture as a continuum in motion was implied in this line of thought. To accomplish this identity of form, the total stress tensor, the total heat flux vector, and the specific energy supply had to be specially defined and did not equal the sum of the corresponding quantities for the constituents. The energy of the mixture was, however, equal to the sum of energies of the constituents. On the other hand, Green postulated the additive property of stress, heat flux, and the energy supply and derived the balance laws from the frame invariance of a rate of energy equality. The energy density of the mixture was seen to be different from the sum of energy densities of the constituents. This was attributed to interaction between the constituents. Green established a relationship between these quantities. Recent work by Gurtin, Oliver, Williams, and Sampaio uses a different approach. The equations of balance of momentum and energy they obtain differ from those of Truesdell and Kelly. Bowen pointed out that for the case of single temperature mixtures, explicit use of an energy equation for the constituents is not needed. Chao noted that combining the balance equations of constituents to obtain the balance equations of the mixture can lead to errors. He cited the absence of inertial coupling forces in the momentum equations of the constituents. However, his own work has been criticized by several investigators.

Green would accept Truesdell's equations as correct but with different interpretations for some of the quantities which occur in these equations. These interpretations are of special significance if boundary conditions on the surface of the mixture involve total stress and total heat flux. Morland supported the additivity of partial stresses on the ground that tractions are additive and Cauchy's stress principle should hold for total stress and total traction as well as for the constituents. In Green's theory, the equations of mass and momentum balance are derived from the material frame invariance of a

rate of energy equality. In another discussion the heat fluxes and the energy supply were assumed to be additive. Green did not define the internal energy and the energy supply for each constituent. It was considered unnecessary for a complete general theory. Later, Green made the role of interactions between constituents explicit by writing the rate of energy equality for each constituent.

Bowen postulated the point form of the rate of energy equality and pointed out the effect of certain approximations. The interpretation of quantities appearing in his equations is quite different from that of similar quantities in Green's theory because the two formulations are based on different definitions for the quantities associated with the mixture in terms of those for the constituents rather than due to any approximation.

Several investigators have used the basic concepts introduced by Truesdell and Green. Most of these efforts aim at simplifying the description of motion for certain special cases. For instance, in granular porous media, the total deformation can be viewed as made up of two parts; one related to deformation of the solid particles and the other to their rearrangement i.e. changes in pore geometry. Volume fraction theories for compressible materials introduce the volume fractions as additional variables.

Fukuo used the equations of mass balance to set up equations in terms of volume fractions of the constituents and specialized these for incompressible solid skeleton and also incompressible fluid. This was based on Gibson's approach of referring to the fixed set of particles in the reference configuration and combined with Darcy's law this may be regarded as a generalization of Gibson's equation. Hsieh considered a porous solid saturated with an incompressible fluid and undergoing small deformations. For this condition, considering a unit volume in the undeformed state, they calculated the fluid content change. Noting that for continued saturation the rate of change in fluid content must equal the divergence of relative velocity, the time derivative of the change in fluid content was calculated to set up a relation between the rate of porosity change, the rate of volumetric strain and the relative velocity vector. These quantities, in this theory for incompressible fluids and no thermal or chemical effects cannot, therefore, be treated as independently variable. For compressible fluids, Morland proposed constitutive equations for porosity. Hsieh referred to this equation as the compatibility condition of volume change of a fluid-saturated porous medium. Goodman postulated the equation of energy balance for a porous material. This equation admitted an additional degree of freedom, viz., the volume fraction. A kinetic energy term was associated with the rate of change of porosity. Similarly, rate of work terms were associated with the rate of change of porosity over volumes and surfaces using generalized forces. Goodman also postulated an equation of motion, called the equation of balance of equilibrated forces and wrote a local form of energy equality.

In most theories of mixtures, deformation is referred to an initial

configuration for each constituent and motion to the place coordinates. The equations of balance are written for a fixed volume in space. This appears to be the correct approach for mixtures of fluids but may not be convenient for multiphase mixtures. The notion of motion of a mixture as a whole is often introduced. Indeed, Truesdell would require the form of the equations of balance for the mixture to be the same as for a single material. We note that for the case of no relative motion between the constituents, the mixture will have motion and deformation as a material body. For that case the development of equations of motion for the mixture is meaningful. However, if relative motion is present, the mixture does not satisfy the axiom of continuity and its corollary, the principle of impenetrability. Hence, the "third metaphysical principle" stated by Truesdell appears to be irrelevant. We note too that the sum of internal energies of the constituents does not equal the internal energy of the mixture. The balance equations due to Truesdell and to Green have similar form and are essentially equivalent but the quantities appearing in the two sets have different interpretations based upon the relationships postulated between the quantities associated with the constituents and with the mixture. The equations of energy balance contain scalar products of corresponding quantities. This indicates the quantities for which constitutive relationships would be required. Some investigators, considering the special problem of flow through deformable porous solids, have attempted to write the balance equations in terms of relative motion and porosity, which is essentially a measure of relative deformation. It would appear that a theory based upon balance equations written for a reference set of particles of the porous solid would be the most appropriate for this case. For the one-dimensional case, Gibson developed such a theory for the quasi-static problem. Work was started to extend this theory to include inertia effects and a generalization to three-dimensional problems.

Because the mixture cannot be viewed as a continuum in motion, it appears inappropriate to define energy functions on the 'mixture' consisting of centers of mass. This implies that in setting up constitutive relationships for the mixture, one cannot invoke the existence of an energy function. Hence, the constitutive relations need not be restricted to a certain class.

Detailed discussion of theories of mixtures is given in item 3 in section 4 above. Certain portions of the problem were discussed in items 6 and 9 of the same section.

## 7.2 Finite Element Studies.

Results for one-dimensional analysis, for which exact solutions are available, showed that the conventional time-domain integration procedures found to be quite effective for single material problems are not reliable for the coupled problem of dynamics of saturated soils. It is necessary, therefore, to investigate the time integration procedures carefully to identify and implement stable schemes relatively insensitive to the parameters selected by the researcher. The results should not be sensitive to the choice

of time-domain integration parameters. Also, as the time step size is reduced, the approximate solution should converge.

Variational formulations of Biot's theory were developed to construct a basis for alternative finite element approaches to the problem. For nonlinear problems, an incremental approach is necessary. For this case a quasi-linearized form of the equation has to be used to develop a variational formulation.

Parametric studies of the mass coupling effect show that the soil displacements are not sensitive to the degree of coupling but the pattern of pore pressure in the time domain could be affected significantly. Further studies in this direction are apparently needed.

### 7.3 Experimental Verification of Theories.

A fine to medium grained sand was chosen as the material to be used in the experimental studies. A program of static tests aimed at identifying basic material properties was performed. A description of the test procedures and results is presented in item 13 of section 4 above. The program of liquefaction experiments has been started. Particular emphasis has been placed on laboratory techniques for proper preparation of the necessary sand samples. Suitable methods for de-airing the approximately 50 kilograms of sand required to build a sample have been developed. Procedures for placing the material at a predetermined density have been studied and samples with a high degree of uniformity can be repeatedly achieved. Instrumentation has been calibrated under both cyclic and static loading conditions. Different types of pore-pressure gage applications have been studied. Initial shaking table tests have been designed to identify most reliable and sensitive instrumentation. Details of the test apparatus and its calibration are contained in the report listed as item 14 in Section 4 above.

## 8. ADDITIONAL COMMENTS AND OTHER WORK, AT THE OHIO STATE UNIVERSITY, RELATED TO THE RESEARCH PROGRAM

The overall problem of motion and stability of fluid-saturated soil systems is extremely complex. Reliable analysis of saturated soil deposits subjected to dynamic loads involves the following three steps:

1. Correct formulation of the equations of dynamic equilibrium.
2. Correct representation of material behavior.
3. Exact or approximate solution of the problem.

Work needs to be done on all these items. At the Ohio State University, a considerable effort, some of it not supported by the AFOSR, is currently under way under the supervision of the principal investigator. Related work includes the following graduate research:

1. Analysis of Wave Scattering in Linear Elastic Soils. Finite element methods and boundary element procedures are being investigated for solution of this problem. Current techniques are based on surrounding the scatterer with an imagined cylindrical surface. Outside this surface, the wave consists of the incident and the reflected components. Inside the surface is the refracted (transmitted) component. The solution process uses the analytical solution for the external domain and a finite element solution for the interior. The two solutions are matched at a discrete number of points on the imaginary surface. At the Ohio State University, the Principal Investigator has been supervising further development of these techniques. Specifically, parametric studies have been carried out to determine the sensitivity of the approximate solution to the choice of the number of source points, distance of the imaginary surface etc. Enforcement of continuity all along the imagined surface instead of only a discrete number of point is expected to result in better accuracy. The procedures already developed for harmonic waves are being extended to the transient case. This work currently being done for homogeneous linearly-elastic media could possibly be extended to analysis of wave propagation in fluid-saturated elastic as well as elastic-plastic systems. This work should be useful in solving problems of dynamic soil/structure interaction. Additional effort is being initiated to study the interference between structures.

2. In finite element procedures it has been difficult to obtain continuous stress fields in the approximations. Investigations at the Ohio State University have shown that mixed methods are not efficient. Other methods, e.g., Loubgniac's, using not only the parameters associated with the nodal points defining the geometry of an element but also including several 'neighboring' nodal points, are being investigated. Work on extending Loubgniac's method to two-dimensional linear elastic stress analysis has shown good results but is very expensive. Additional investigations on continuous stress elements based on higher order interpolations are in progress.

3. Fluid-saturated soils can undergo large deformations under dynamic loads. Most of the current procedures are based on the small deformation theory. At the Ohio State University, work on development of methods of representation of soil behavior under large deformations and to develop finite element solution procedures which can satisfactorily allow for such material models as well as large deformations is continuing.

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